CHAPTER (3)

FLUID STATICS

SOLVED PROBLEMS

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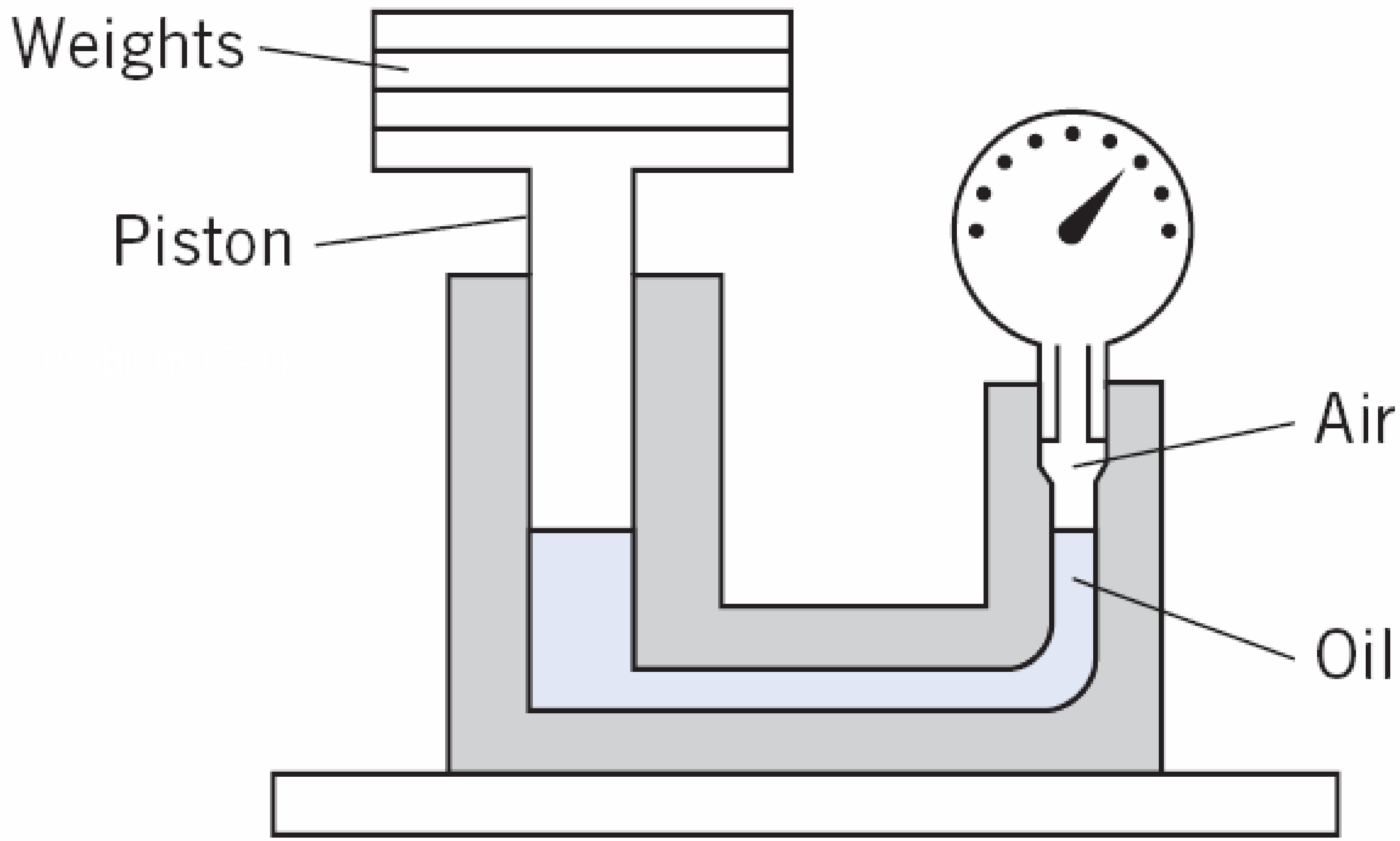
Problem (3.1)



PROBLEM 3.1

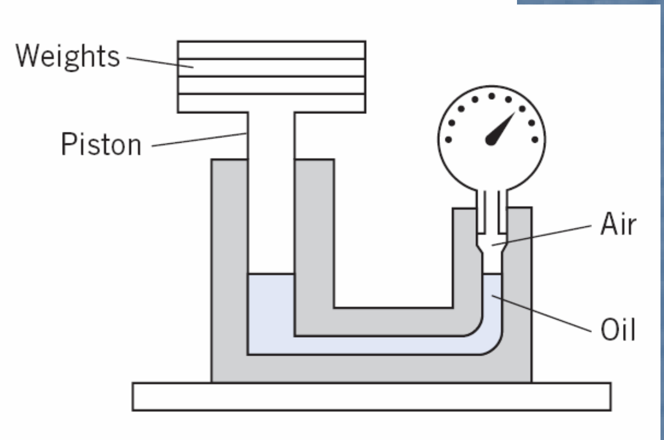
Situation: A Crosby gage tester is applied to calibrate a pressure gage.
A weight of 140 N results in a reading of 200 kPa.
The piston diameter is 30 mm.

Find: Percent error in gage reading.

APPROACH

Calculate the pressure that the gage should be indicating (true pressure). Compare this true pressure with the actual pressure.

ANALYSIS



True pressure

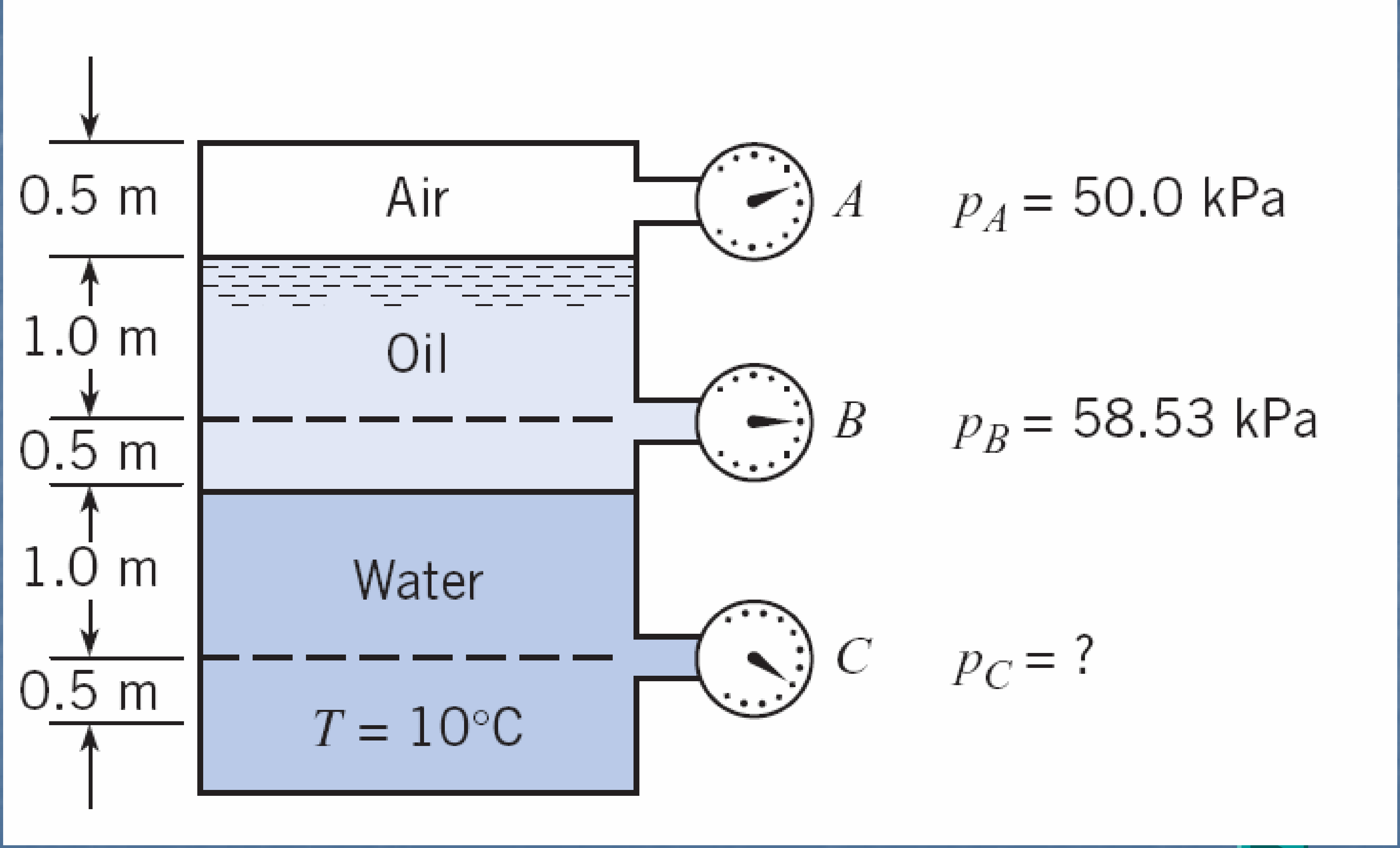
$$\begin{aligned} p_{\text{true}} &= \frac{F}{A} \\ &= \frac{140\,\text{N}}{(\pi/4 \times 0.03^2)\ \text{m}} \\ &= 198,049\,\text{kPa} \end{aligned}$$

Percent error

$$\begin{aligned} \%\ \text{Error} &= \frac{(p_{\text{recorded}} - p_{\text{true}})\,100}{p_{\text{true}}} \\ &= \frac{(200-198)\,100}{198} \\ &= 1.0101\% \end{aligned}$$

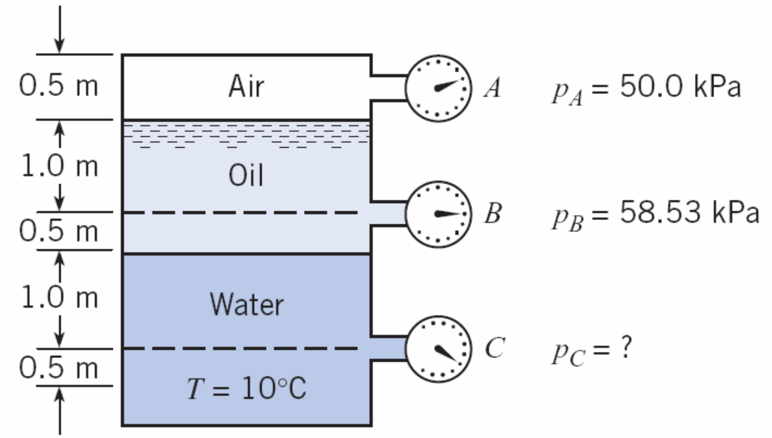
$$\boxed{\%\ \text{Error} = 1.01\%}$$

Problem (3.7)


0.5 m
1.0 m
0.5 m
1.0 m
0.5 m
Air
Oil
Water
T = 10°C
A
B
C
p_A = 50.0 kPa
p_B = 58.53 kPa
p_C = ?

PROBLEM 3.7

Situation: A closed tank with Bourdon-tube gages tapped into it is described in the problem statement.



Find:
(a) Specific gravity of oil.
(b) Pressure at C.

APPROACH

Apply the hydrostatic equation.

ANALYSIS

Hydrostatic equation (from oil surface to elevation B)

$$
\begin{aligned}
p_A + \gamma z_A &= p_B + \gamma z_B \\
50,000 \text{ N/m}^2 + \gamma_{\text{oil}} (1 \text{ m}) &= 58{,}530 \text{ N/m}^2 + \gamma_{\text{oil}} (0 \text{ m}) \\
\gamma_{\text{oil}} &= 8530 \text{ N/m}^2
\end{aligned}
$$

Specific gravity

$$S = \frac{\gamma_{\text{oil}}}{\gamma_{\text{water}}} = \frac{8530 \text{ N/m}^2}{9810 \text{ N/m}^2}$$

$$\boxed{S_{\text{oil}} = 0.87}$$

Hydrostatic equation (in water)

$$p_c = (p_{\text{btm of oil}}) + \gamma_{\text{water}} (1 \text{ m})$$

Fluid Statics

Hydrostatic equation (in oil)

$$p_{\text{btm of oil}} = (58,530\,\text{Pa} + \gamma_{\text{oil}} \times 0.5\,\text{m})$$

Combine equations

$$\begin{aligned} p_c &= (58,530\,\text{Pa} + \gamma_{\text{oil}} \times 0.5\,\text{m}) + \gamma_{\text{water}}\,(1\,\text{m}) \\ &= (58,530 + 8530 \times 0.5) + 9810\,(1) \\ &= 72,605\ \text{N/m}^2 \end{aligned}$$

$$\boxed{p_c = 72.6\ \text{kPa}}$$

Fluid Statics

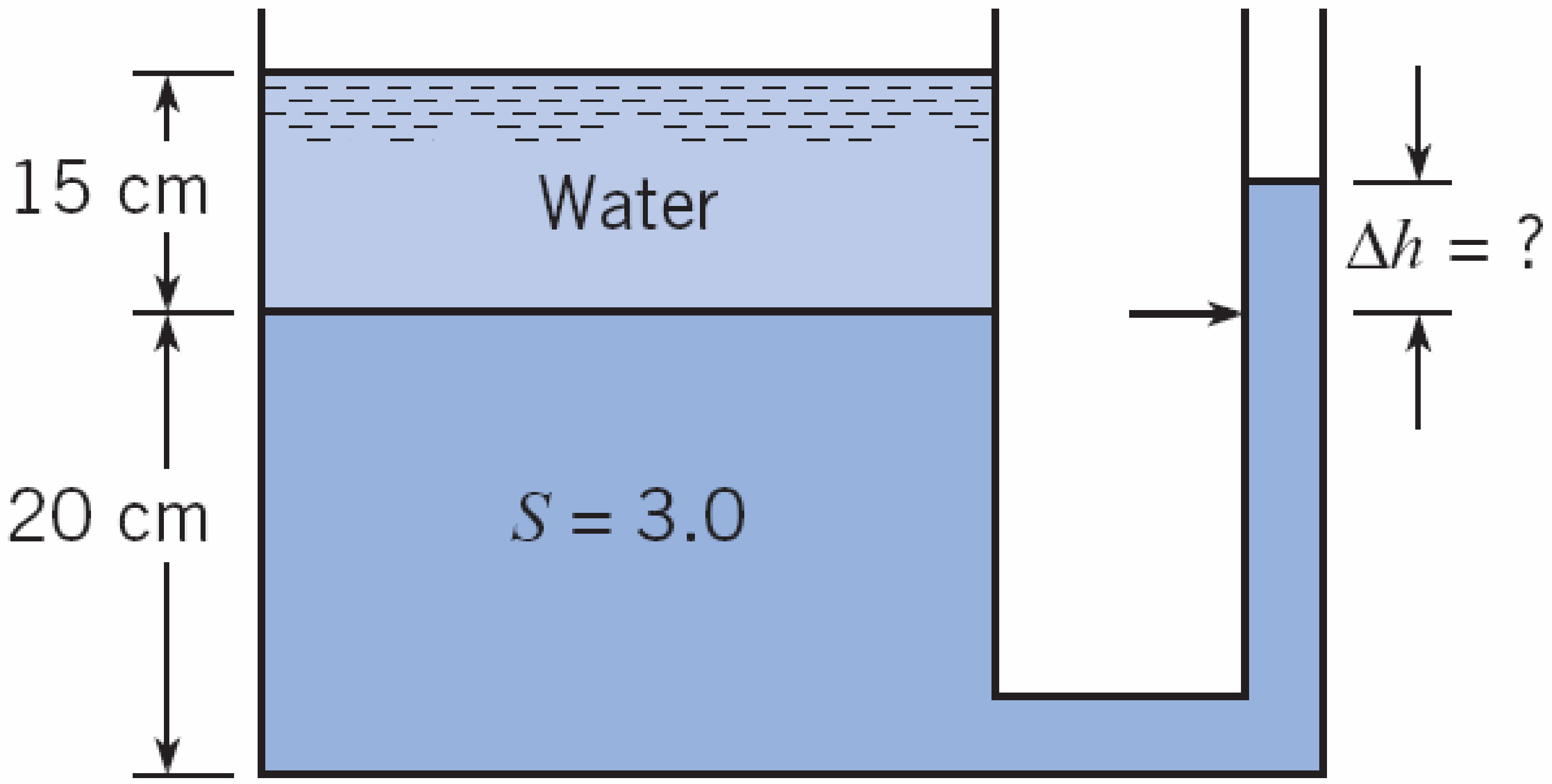
$$
\begin{aligned}
p_2 + \gamma z_2 &= p_1 + \gamma z_1 \\
p_2 &= p_1 + (S\gamma_{\text{water}})(z_1 - z_2) \\
&= 1.592 \times 10^5 \text{ Pa} + \left(0.85 \times 9810 \text{ N/ m}^3\right)(-2 \text{ m}) \\
&= 1.425 \times 10^5 \text{ Pa}
\end{aligned}
$$

Equilibrium (piston 2)

$$
\begin{aligned}
F_2 &= p_2 A_2 \\
&= \left(1.425 \times 10^5 \text{ N/ m}^2\right)\left(\frac{\pi (0.1 \text{ m})^2}{4}\right) \\
&= 1119 \text{ N}
\end{aligned}
$$

$$
\boxed{F_2 = 1120 \text{ N}}
$$

Problem (3.15)



PROBLEM 3.15

Situation: A tank fitted with a manometer is described in the problem statement.

Find: Deflection of the manometer.(Δh)

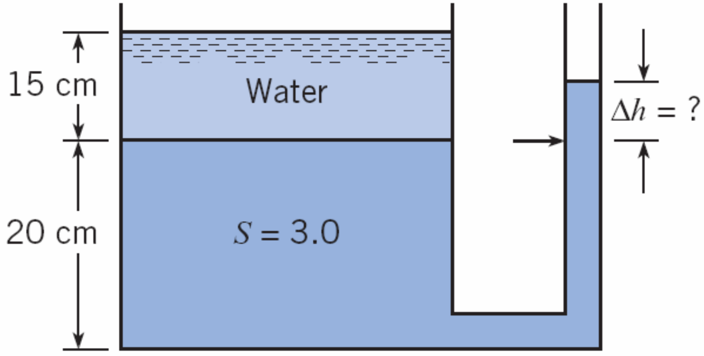
APPROACH

Apply the hydrostatic principle to the water and then to the manometer fluid.

ANALYSIS

Hydrostatic equation (location 1 is on the free surface of the water; location 2 is the interface)

$$\begin{aligned} \frac{p_1}{\gamma_{\text{water}}} + z_1 &= \frac{p_2}{\gamma_{\text{water}}} + z_2 \\ \frac{0\,\text{Pa}}{9810\,\text{N/m}^3} + 0.15\,\text{m} &= \frac{p_2}{9810\,\text{N/m}^3} + 0\,\text{m} \\ p_2 &= (0.15\,\text{m})\left(9810\,\text{N/m}^3\right) \\ &= 1471.5\,\text{Pa} \end{aligned}$$



Hydrostatic equation (manometer fluid; let location 3 be on the free surface)

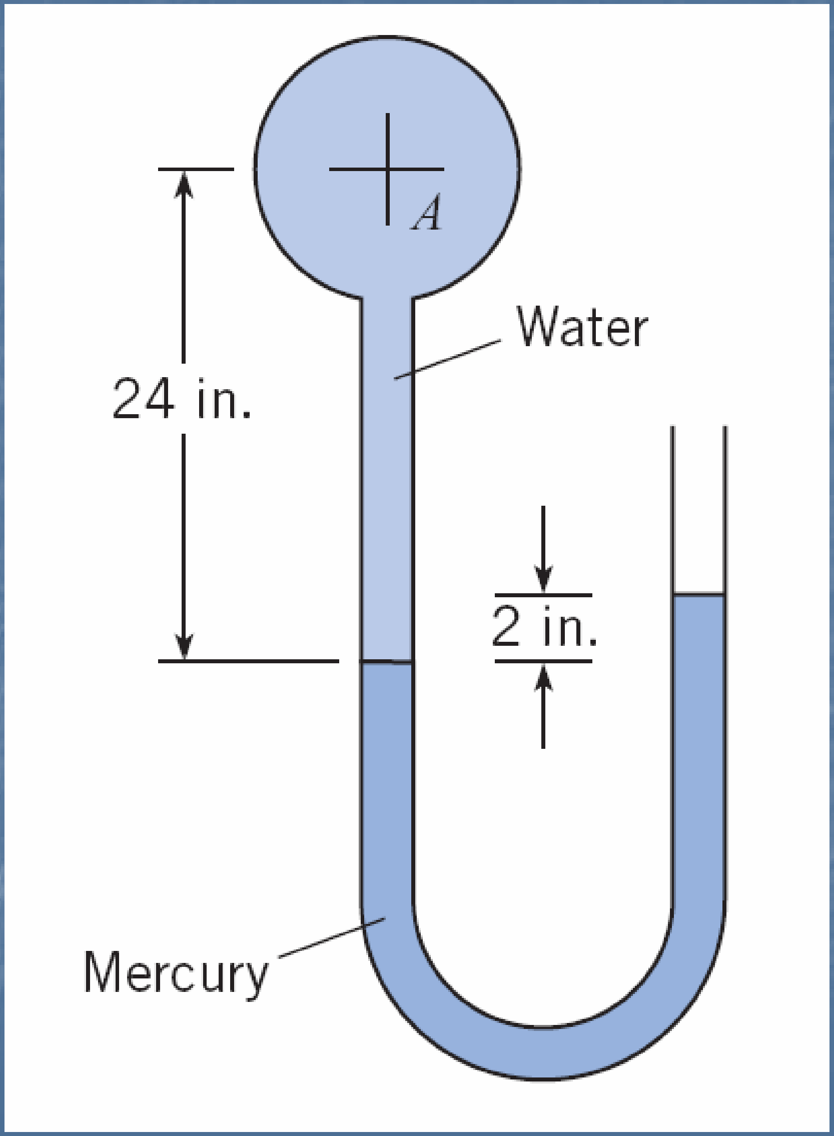
$$\begin{aligned} \frac{p_2}{\gamma_{\text{man. fluid}}} + z_2 &= \frac{p_3}{\gamma_{\text{man. fluid}}} + z_3 \\ \frac{1471.5\,\text{Pa}}{3\left(9810\,\text{N/m}^3\right)} + 0\,\text{m} &= \frac{0\,\text{Pa}}{\gamma_{\text{man. fluid}}} + \Delta h \end{aligned}$$

Solve for Δh

$$\begin{aligned} \Delta h &= \frac{1471.5\,\text{Pa}}{3\left(9810\,\text{N/m}^3\right)} \\ &= 0.0500\,\text{m} \end{aligned}$$

$$\boxed{\Delta h = 5.00\ \text{cm}}$$

Problem (3.28)



Fluid Statics

PROBLEM 3.28

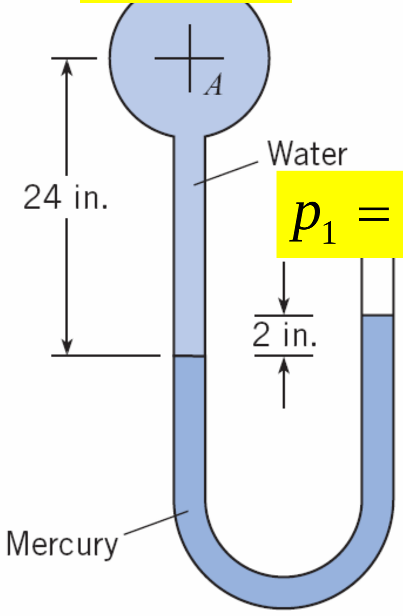
Situation: A pipe system is described in the problem statement.

Find: Gage pressure at pipe center.

Find pressure at center of pipe (A)=?

$p_A = p_f$

$p_1 = p_i$



APPROACH

Apply the manometer equation.

ANALYSIS

Manometer equation (from A to the open end of the manometer

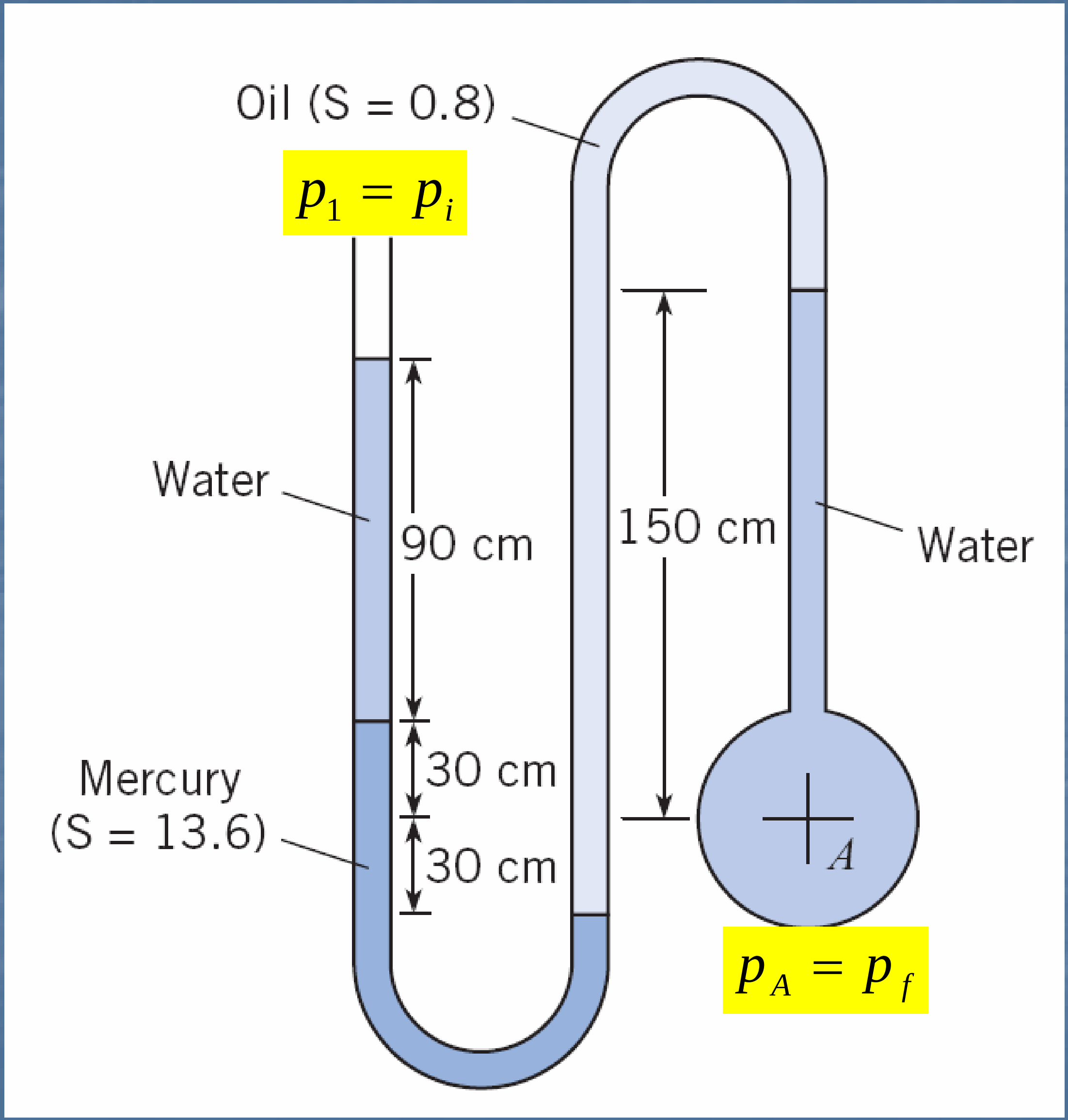
$$p_A + (2.0 \text{ ft})(62.3 \text{ lbf/ft}^3) - (2/12 \text{ ft})(847 \text{ lbf/ft}^3)$$

$$p_A = -124.6 \text{ lbf/ft}^2 + 141.2 \text{ lbf/ft}^2 = +16.6 \text{ lbf/ft}^2$$

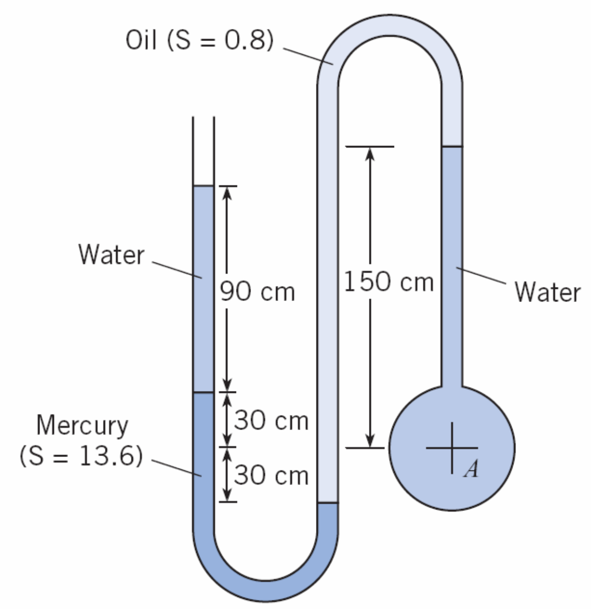
$$\boxed{p_A = +0.12 \text{ psi}}$$

Problem (3.39)

Find pressure at center of pipe (A)=?



Fluid Statics



PROBLEM 3.39

Situation: A pipe system is described in the problem statement.

Find: Pressure at center of pipe A.

ANALYSIS

Manometer equation

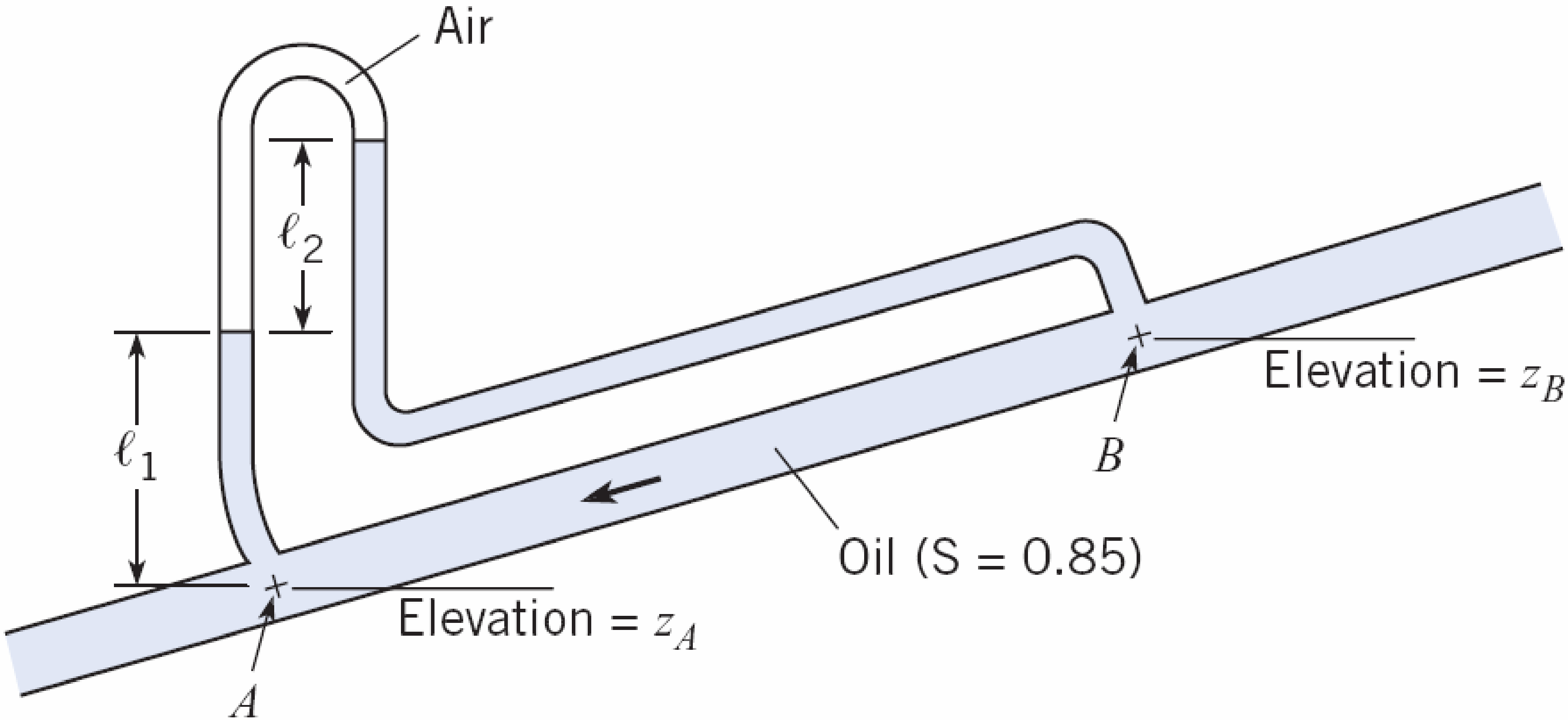
S.G

$\gamma_{water} = \rho g = 1000 \times 9.81$

$$p_A = (0.9 + 0.6 \times 13.6 - 1.8 \times 0.8 + 1.5)9,810 = 89,467 \text{ Pa}$$

$$\boxed{p_A = 89.47 \text{ kPa}}$$

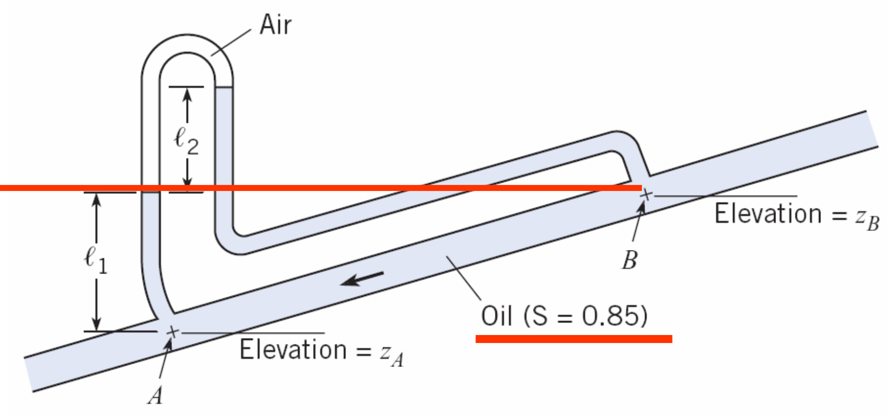
Problem (3.40)



PROBLEM 3.40

Situation: A pipe system is described in the problem statement.

Find: (a) Difference in pressure between points A and B.
(b) Difference in piezometric head between points A and B.



APPROACH

Apply the manometer equation.

ANALYSIS

Manometer equation

$$p_A - (1\,\text{m})\left(0.85 \times 9810\,\text{N/m}^3\right) + (0.5\,\text{m})\left(0.85 \times 9810\,\text{N/m}^3\right) = p_B$$
$$p_A - p_B = 4169\,\text{Pa}$$

$$\boxed{p_A - p_B = 4.169\ \text{kPa}}$$

$$L_1 = 1m$$
$$L_2 = 50cm$$
$$Z_B - Z_A = 11 - 10 = 1m$$

Piezometric head

$$\begin{aligned} h_A - h_B &= \left(\frac{p_A}{\gamma} + z_A\right) - \left(\frac{p_B}{\gamma} + z_B\right) \\ &= \frac{p_A - p_B}{\gamma} + (z_A - z_B) \\ &= \frac{4169\,\text{N/m}^2}{0.85 \times 9810\,\text{N/m}^3} - 1\,\text{m} \\ &= -0.5\ \text{m} \end{aligned}$$

$$\boxed{h_A - h_B = -0.50\ \text{m}}$$

Problem (3.65)

$\alpha = 60^\circ$

PROBLEM 3.65

Situation: A butterfly valve is described in the problem statement.

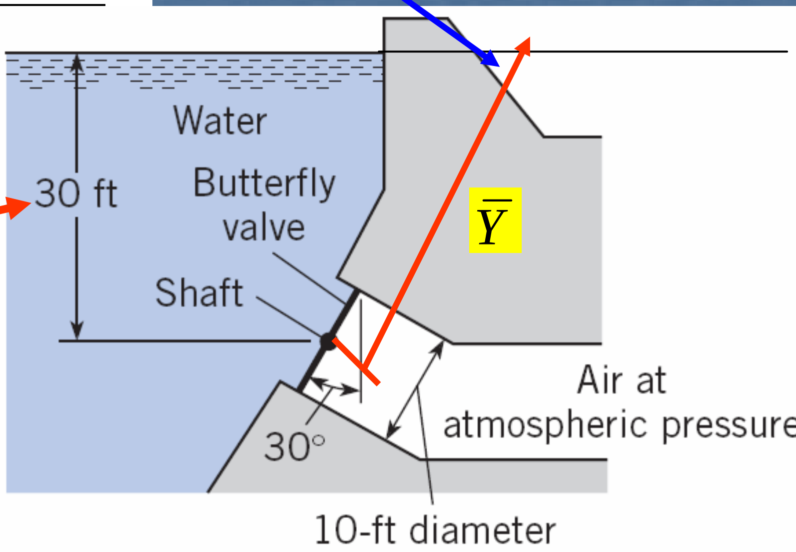
Find: Torque required to hold valve in position.

ANALYSIS

$Note: h_{C.G} = \bar{y}\sin\alpha = 30\,ft$

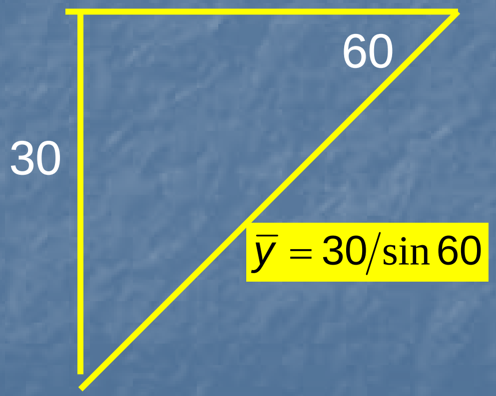
Hydrostatic force

$$
\begin{aligned}
F &= \bar{p}A = \bar{y}\gamma A \quad (\sin\alpha)\\
&= (30\text{ ft} \times 62.4\text{ lb/ft}^3)(\pi \times D^2/4)\text{ ft}^2)\\
&= (30 \times 62.4 \times \pi \times 10^2/4)\text{ lb}\\
&= 147,027\text{ lb}
\end{aligned}
$$



Center of pressure

$$
\begin{aligned}
y_{cp} - \bar{y} &= I/\bar{y}A\\
&= (\pi r^4/4)/(\bar{y}\pi r^2)\\
&= (5^2/4)/(30/.866)\\
&= 0.1804\text{ ft}
\end{aligned}
$$



Torque

$$
\text{Torque} = 0.1804 \times 147,027
$$

$$
\boxed{T = 26,520\text{ ft-lbf}}
$$

Problem (3.65) - HW

PROBLEM 3.78

Situation: A curved surface is described in the problem statement.

Find: (a) Vertical hydrostatic force.

(b) Horizontal hydrostatic force.

(c) Resultant force.

$L = 1m$

ANALYSIS

$$F_V = 1 \times 9,810 \times 1 \times +(1/4)\pi \times (1)^2 \times 1 \times 9,810$$

$$\boxed{F_V = 17,515 \text{ N}}$$

$$x = M_0/F_V$$

$$= 1 \times 1 \times 1 \times 9,810 \times 0.5 + 1 \times 9,810 \times \int_0^1 \sqrt{1-x^2}x dx/17,515$$

$$= \boxed{0.467 \text{ m}}$$

$$F_H = \bar{p}A$$

$$= (1+0.5)9,810 \times 1 \times 1$$

$$\boxed{F_H = 14,715 \text{ N}}$$

$$y_{cp} = \bar{y} + \bar{I}/\bar{y}A$$

$$= 1.5 + (1 \times 1^3)/(12 \times 1.5 \times 1 \times 1)$$

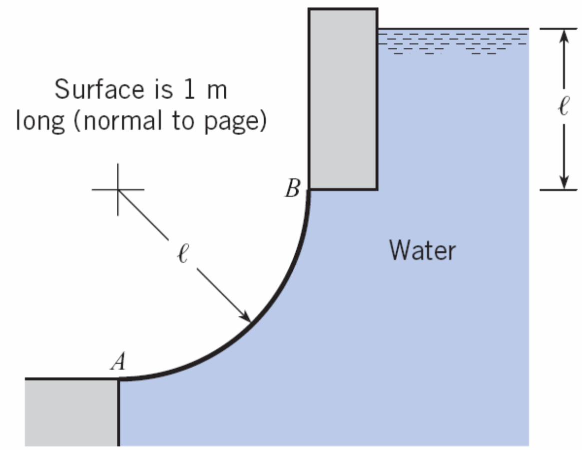
$$\boxed{y_{cp} = 1.555 \text{ m}}$$

$$F_R = \sqrt{(14,715)^2 + (17,515)^2}$$

$$\boxed{F_R = 22,876 \text{ N}}$$

$$\tan\theta = 14,715/17,515$$

$$\boxed{\theta = 40°2'}$$



$$F_v = \gamma \bar{y} \sin\alpha A = (9810)(1)(1\times 1) = 9810N$$

$$W = \gamma V_{ABC} = 9810 \times \frac{1}{4}\pi \times 1^2 = 7704.76N$$

$$F_y = 9810 + 7704.76 = 17514.76N$$

$$x_{cp}F_y = (0.5 \times F_V) + (W \times \frac{4r}{3\pi})$$

$$x_{cp} = \frac{(9810 \times 0.5) + (7704.76 \times \frac{4\times 1}{3\pi}}{17514.76} = 0.467$$

Problem (3.82)

PROBLEM 3.82

Situation: A dome below the water surface is described in the problem statement.

Find: Magnitude and direction of force to hold dome in place.

ANALYSIS

$h_{C.G} = \bar{y}\sin\alpha = y_1 + y_2/2 = 1 + 1 = 2$

$y_1 = 1\,m$

$y_2 = 2\,m$

$$\begin{aligned} F_H &= (1+1)9810 \times \pi \times (1)^2 \\ &= 61,640 \text{ N } = 61.64 \text{ kN} \end{aligned}$$

This 61.64 kN force will act horizontally to the left to hold the dome in place.

$\bar{y} = 1 + 1$

$I = \dfrac{\pi r^4}{4}$

$$\begin{aligned} (y_{cp} - \bar{y}) &= I/\bar{y}A \\ &= (\pi \times 1^4/4)/(2 \times \pi \times 1^2) \\ &= 0.125 \text{ m} \end{aligned}$$

$F_v = \gamma V_{dome}$

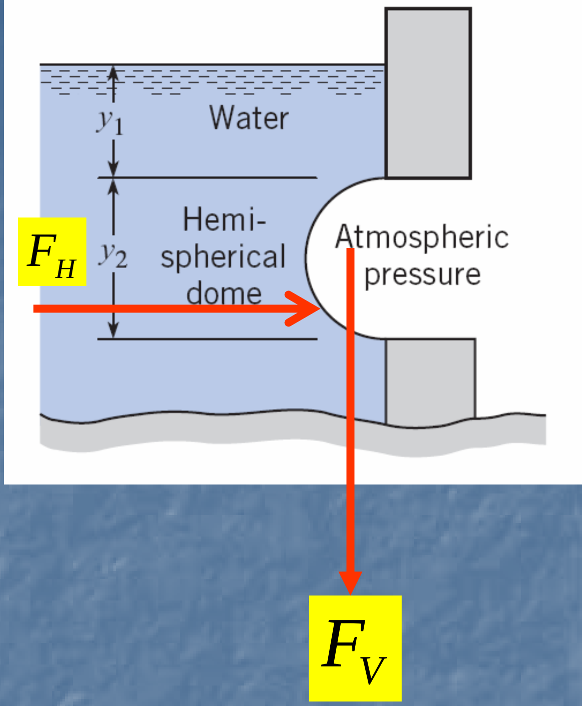
The line of action lies 0.125 m below the center of curvature of the dome.

$(V_{dome})_{Sphere} = \dfrac{4\pi r^3}{3}$

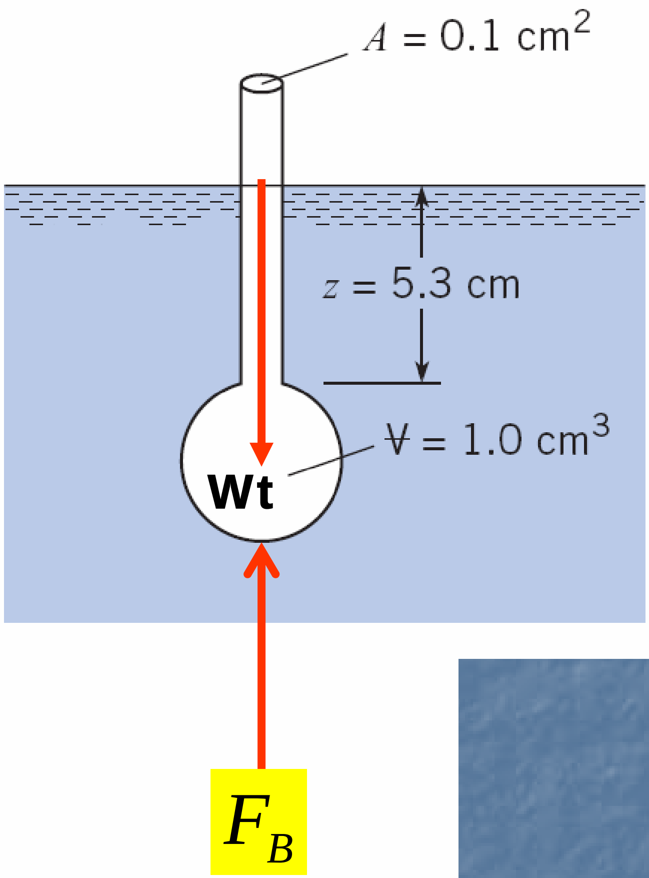
$$\begin{aligned} F_V &= (1/2)(4\pi \times 1^3/3)9,810 \\ &= 20,550 \text{ N} \end{aligned}$$

$$\boxed{F_V = 20.55 \text{ kN}}$$

To be applied downward to hold the dome in place.



Problem (3.101)



PROBLEM 3.101

Situation: A hydrometer is described in the problem statement.

Find: Weight of hydrometer.

ANALYSIS

Volume of immersed part

$$F_{\text{buoy.}} = W\,.$$

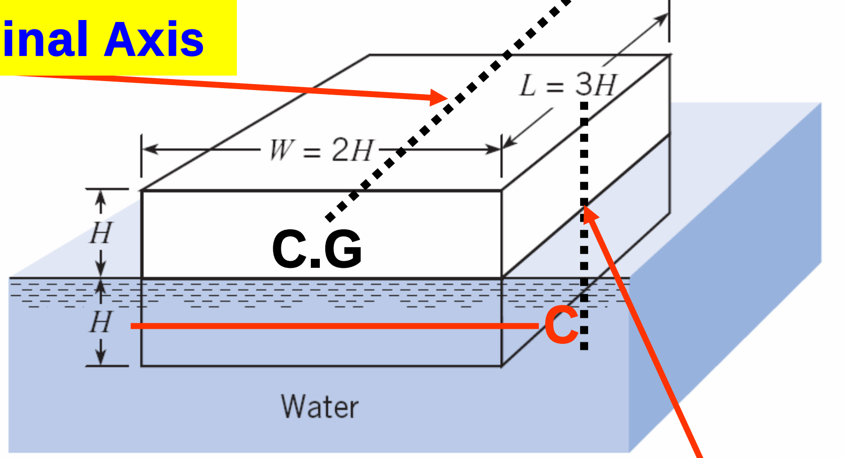
$$V\!\!\!- \gamma_W = W$$

$$(1\ \text{cm}^3 + (5.3\ \text{cm})(0.01\ \text{cm}^2))(0.1^3)\ \text{m}^3/\text{cm}^3(\gamma_W) = W\,.$$

$$(1.53\ \text{cm}^3)(10^{-6}\ \text{m}^3/\text{cm}^3)(9810\ \text{N/m}^3) = W.$$

$$\boxed{W = 1.50 \times 10^{-2}\ \text{N}}$$

Problem (3.101)



PROBLEM 3.110

Situation: A floating block is described in the problem statement.

Find: Stability.

ANALYSIS

Analyze longitudinal axis

$$\begin{aligned} GM &= I_{00}/V - CG \\ &= (3H(2H)^3/(12 \times H \times 2H \times 3H)) - H/2 \\ &= -H/6 \end{aligned}$$

Not stable about longitudinal axis.

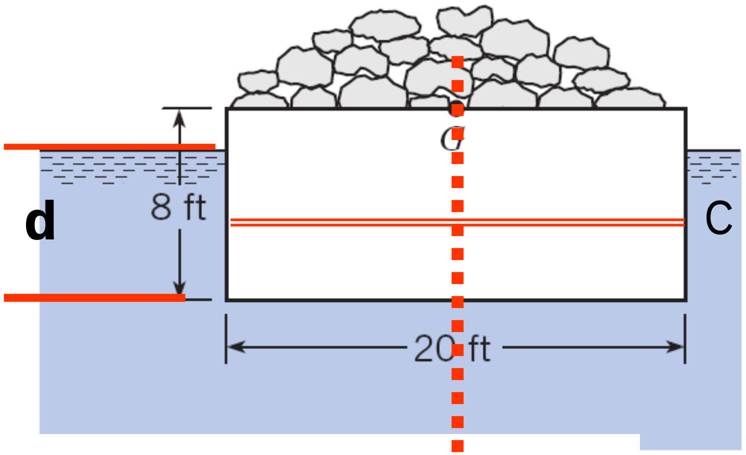
Analyze transverse axis.

$$\begin{aligned} GM &= (2H \times (3H)^3/(12 \times H \times 2H \times 3H)) - 3H/4 \\ &= 0 \end{aligned}$$

Neutrally stable about transverse axis.

Not stable

Problem (3.105)



PROBLEM 3.106

Situation: A barge is described in the problem statement.

Find: Stability of barge.

Axis of Symmetry

ANALYSIS

Wt   L

$$\text{Draft} = 400,000/(50 \times 20 \times 62.4) = 6.41 \text{ ft} < 8 \text{ ft}$$

$$\begin{aligned} GM &= I_{00}/V\!\!\!- - CG \\ &= \left[(50 \times 20^3/12)/(6.41 \times 50 \times 20)\right] - (8 - 3.205) \\ &= 0.40 \text{ ft} \end{aligned}$$

Will float stable

END OF QUESTIONS